

## Units and Dimensions

The quantities which can be measured by an instrument and by means of which we can describe the laws of physics are called physical quantities. There are two types of physical quantities.

1. Fundamental quantities

2. Derived quantities

### Fundamental Quantities (Base quantities) :

A set of minimum number of quantities required to give a consistent and unambiguous description of all other quantities of physics are called fundamental or base quantities. The fundamental quantities should be independent of each other. In S.I. there are seven fundamental quantities. They are mass, length, time, temperature, luminous intensity, electric current and number of particles.

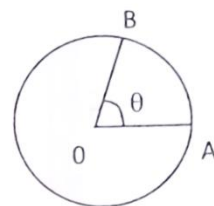
### Supplementary Quantities :

There are two more quantities to supplement the fundamental quantities called supplementary quantities namely plane angle and solid angle. A phenomena relating angular motion can be expressed in two different ways e.g. a particle moves from A to B along an arc.

**First statement :** A travels a distance  $\ell$  on the circumference of a circle of radius  $r$ .

**Second statement :** A travels through an angle  $\theta$  on the circumference of a circle of radius  $r$ .

These ways are dependent as  $\theta = \frac{\ell}{r}$ . So these angular quantities are called supplementary quantities.



### Derived quantities :

These quantities are derived from fundamental quantities e.g. velocity, force, momentum etc.

**Note :** Any set of independent quantities can be chosen as basic quantities by which all other physical quantities can be derived.

### Unit :

To measure a physical quantity, we have to compare it with a certain basic, arbitrarily chosen, internationally accepted reference standard. This reference standard is called a unit.

The international system (S.I) contains three classes of units. They are:

1. Fundamental (base)units

2. Supplementary units

3. Derived units

#### 1. Fundamental units :

These units are reference standard of fundamental physical quantities.

|    | Quantity            | Unit     | Symbol |
|----|---------------------|----------|--------|
| 1. | Length              | metre    | m      |
| 2. | Mass                | kilogram | kg     |
| 3. | Time                | second   | s      |
| 4. | Luminous intensity  | candela  | cd     |
| 5. | Temperature         | Kelvin   | K      |
| 6. | Electric current    | ampere   | A      |
| 7. | Number of particles | mole     | mol    |

#### 2. Supplementary units :

We have two supplementary quantities. They are plane angle and solid angle. Their units are radian (symbol is 'rad') and steradian (symbol is 'sr') respectively.

### Derived units :

Derived units are formed by combining fundamental units and are used as reference standard for derived quantities e.g. unit of force is  $\text{kg-m/s}^2 = \text{newton (N)}$ .

### S.I prefixes and multiplication factors :

| Multiplication factors | Prefix | Symbol |
|------------------------|--------|--------|
| $10^{18}$              | exa    | E      |
| $10^{15}$              | peta   | P      |
| $10^{12}$              | tera   | T      |
| $10^9$                 | giga   | G      |
| $10^6$                 | mega   | M      |
| $10^{-3}$              | milli  | m      |
| $10^{-6}$              | micro  | $\mu$  |
| $10^{-9}$              | nano   | n      |
| $10^{-12}$             | pico   | p      |
| $10^{-15}$             | femto  | f      |
| $10^{-18}$             | atto   | a      |

For lengths, other units are also used

$$1 \text{ angstrom } (\text{\AA}) = 10^{-10} \text{ m}$$

$$1 \text{ Parsec} = 3.26 \text{ light years}$$

$$1 \text{ Micron} = 10^{-6} \text{ m}$$

$$1 \text{ light year} = 9.46 \times 10^{15} \text{ m}$$

$$\text{Solar constant} = 1340 \text{ Wm}^{-2}$$

$$1 \text{ barn} = 10^{-28} \text{ m}^2$$

$$1 \text{ fermi} = 10^{-15} \text{ m}$$

$$1 \text{ X-ray unit (XU)} = 10^{-13} \text{ m}$$

$$1 \text{ Astronomical unit (AU)} = 1.5 \times 10^{11} \text{ m}$$

$$1 \text{ Shake} = 10^{-8} \text{ s}$$

$$1 \text{ torr} = 1 \text{ mm of Hg} = 133.3 \text{ Pa}$$

$$1 \text{ Chandra Shekar limit} = 1.4 \times \text{mass of Sun}$$

### Points to be remembered while using S.I. units :

1. Even if a unit is named after a person, the unit is not written with capital initial letter.
2. For a unit named after a person, the symbol is a capital letter.
3. The symbols are never expressed in plural form.
4. Full stops and other punctuation marks are not to be written after symbols.

### Dimensions of a Physical Quantity :

"The dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent a derived unit of that quantity."

The seven fundamental or base quantities chosen in SI are called seven dimensions of the physical world. They are usually denoted with square brackets [ ]. Thus, length is represented by [L], mass by [M], time by [T], electric current by [A] or [I], thermodynamical temperature by [K] or [ $\theta$ ], Luminous intensity by [Cd] and amount of substance by [mol].

Note that using the square brackets [ ] round a quantity means that we are dealing with the dimensions of the quantity.

In mechanics, all the physical quantities can be written in terms of the dimensions of [L], [M] and [T]. For example

$$\text{Volume} = \text{length} \times \text{breadth} \times \text{height}$$

$$\therefore \text{Volume} = [L] \times [L] \times [L] = [L^3]$$

$$\text{Mass} = [M]$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Density} = \frac{[M]}{[L^3]}$$

$$\text{Density} = [ML^{-3}T^0]$$



Thus to present density, we have to raise [L] to the power -3 and [M] to the power 1. Therefore, density is said to have negative *three* dimensions in length one dimension of mass.

As unit of time is not required in representing density, we write

Density =  $[ML^{-3}T^0]$ , and say that volume has 1 dimension in mass and zero dimension in time in addition to -3 dimensions in length.

Similarly, for velocity, we write,

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}} = \frac{[L]}{[T]}$$

$$\text{Velocity} = [L^1 T^{-1}] = [M^0 L^1 T^{-1}]$$

Hence the dimensions of velocity are : zero in mass, +1 in length and -1 in time.

Note that in dimensional representation, the magnitudes are not considered. It is the quality of the physical quantity that matters. For example, speed, velocity, initial velocity, final velocity, change in velocity, instantaneous velocity, relative velocity – all are equivalent in this context having dimensions  $[M^0 L^1 T^{-1}]$

### Dimensional Formulae and dimensional Equations :

The expression which shows how and which of the base quantities represent the dimensions of a physical quantity is called the dimensional formula of the given physical quantity.

For example, as deduced above,  $[M^0 L^1 T^{-1}]$  is the dimensional formula of velocity. It reveals that unit of velocity depends on [L] and [T]. It does not depend on [M]. Further, unit of velocity varies directly as unit of length and inversely as unit of time.

An equation obtained by equating a physical quantity with its dimensional formula is called the dimensional equation of the physical quantity.

Dimensional equation of volume (V) is represented as

$$V = [M^0 L^3 T^0]$$

### Dimensional Equations of Some Quantities :

$$(i) \text{ acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

$$(ii) a = \frac{L/T}{T} = LT^{-2} = [M^0 L^1 T^{-2}]$$

$$(iii) \text{ As force} = \text{mass} \times \text{acceleration}$$

$$F = [M] \times [L^1 T^{-2}]$$

$$F = [M^1 L^1 T^{-2}]$$

**Example 1:** Write dimensions of the gravitational constant G, Potential difference, Capacitance, Inductance, Specific heat.

$$\text{Solution : We know that } F = G \frac{m_1 m_2}{r^2} \Rightarrow G = \frac{Fr^2}{m_1 m_2}$$

$$\therefore [G] = \frac{[MLT^{-2}][L^2]}{[M][M]} \Rightarrow [G] = [M^{-1} L^3 T^{-2}]$$

$$\text{Potential difference } V = \frac{\text{work}}{\text{charge}} = \frac{ML^2 T^{-2}}{IT} = ML^2 T^{-3} I$$

$$\text{Capacitance } C = \frac{\text{charge}}{\text{potential difference}} = \frac{\text{charge}}{\frac{\text{work}}{\text{charge}}} = \frac{IT \times IT}{ML^2T^{-2}} = M^{-1}L^{-2}T^4I^2$$

$$\text{Inductance } EMF = L \frac{di}{dt} \Rightarrow \frac{\text{work}}{\text{charge}} = L \frac{\text{current}}{\text{time}} \Rightarrow \frac{ML^2T^{-2}}{IT} = \ell \frac{I}{T} \Rightarrow \ell = ML^2T^{-2}I^{-2}$$

$$\text{Heat} = \text{mass} \times \text{specific heat} \times \text{change in temperature} \Rightarrow ML^2T^{-2} = M \times S \times K \Rightarrow S = L^2T^{-2}K^{-1}$$

**NOTE :** Dimension of a physical quantity will be same, it doesn't depend on which formula we are using for that quantity.

The dimensional formulae of some of the important quantities are derived below and are listed in Table. The SI units of all these quantities are also given in the table.

| Sl. No. | Quantities                          | S.I. unit/symbol                    | Dimensional formula  |
|---------|-------------------------------------|-------------------------------------|--|
| 1.      | Displacement                        | m                                   | [L]  |
| 2.      | Velocity                            | ms <sup>-1</sup>                    | [LT <sup>-1</sup> ]  |
| 3.      | Acceleration                        | ms <sup>-2</sup>                    | [LT <sup>-2</sup> ]  |
| 4.      | Force                               | N                                   | [MLT <sup>-2</sup> ]   |
| 5.      | Work/Energy                         | J                                   | [ML <sup>2</sup> T <sup>-2</sup> ]                                   |
| 6.      | Torque/moment of force              | N-m                                 | [ML <sup>2</sup> T <sup>-2</sup> ]                                   |
| 7.      | Pressure                            | Pascal = Nm <sup>-2</sup>           | [ML <sup>-1</sup> T <sup>-2</sup> ]                                  |
| 8.      | Surface Tension                     | Nm <sup>-1</sup>                    | [MT <sup>-2</sup> ]  |
| 9.      | Density                             | kg m <sup>-3</sup>                  | [ML <sup>-3</sup> ]  |
| 10.     | Momentum                            | kg ms <sup>-1</sup>                 | [MLT <sup>-1</sup> ]   |
| 11.     | Coefficient of Viscosity            | Poiseulle or Pa-s                   | [ML <sup>-1</sup> T <sup>-1</sup> ]                                  |
| 12.     | Power                               | W                                   | [ML <sup>2</sup> T <sup>-3</sup> ]                                   |
| 13.     | Stress                              | Nm <sup>-2</sup>                    | [ML <sup>-1</sup> T <sup>-2</sup> ]                                  |
| 14.     | Angular displacement                | rad                                 | No dimensions  |
| 15.     | Angular Momentum                    | kg m <sup>2</sup> s <sup>-1</sup>   | [ML <sup>2</sup> T <sup>-1</sup> ]                                   |
| 16.     | Relative Density                    | no unit                             | No dimensions  |
| 17.     | Strain                              | no unit                             | No dimensions  |
| 18.     | Specific heat                       | J kg <sup>-1</sup> K <sup>-1</sup>  | [L <sup>2</sup> T <sup>-2</sup> K <sup>-1</sup> ]                    |
| 19.     | Intensity                           | Wm <sup>-2</sup>                    | [MT <sup>-3</sup> ]  |
| 20.     | Planck's constant                   | J-S                                 | [ML <sup>2</sup> T <sup>-1</sup> ]                                   |
| 21.     | Latent Heat                         | J kg <sup>-1</sup>                  | [L <sup>2</sup> T <sup>-2</sup> ]                                    |
| 22.     | Coefficient of thermal conductivity | Wm <sup>-1</sup> K <sup>-1</sup>    | [MLT <sup>-3</sup> K <sup>-1</sup> ]                                 |
| 23.     | Mechanical equivalent of heat       | no unit                             | no dimensions  |
| 24.     | Boltzmann constant                  | J K <sup>-1</sup>                   | [ML <sup>2</sup> T <sup>-2</sup> K <sup>-1</sup> ]                   |
| 25.     | Gas constant                        | J mol <sup>-1</sup> K <sup>-1</sup> | [ML <sup>2</sup> T <sup>-2</sup> K <sup>-1</sup> mol <sup>-1</sup> ] |
| 26.     | Charge                              | C                                   | [AT]   |
| 27.     | Electric potential                  | JC <sup>-1</sup> = V                | [ML <sup>2</sup> T <sup>-3</sup> A <sup>-1</sup> ]                   |
| 28.     | Electric intensity                  | NC <sup>-1</sup>                    | [MLT <sup>-3</sup> A <sup>-1</sup> ]                                 |
| 29.     | Capacitance                         | F                                   | [M <sup>-1</sup> L <sup>2</sup> T <sup>4</sup> A <sup>2</sup> ]      |

|     |   |                     |                        |
|-----|---|---------------------|------------------------|
| 30. | Permittivity of free space ( $\epsilon_0$ ) | $C^2 N^{-1} m^{-1}$ | $[M^{-1}L^{-3}T^4A^2]$ |
| 31. | Dielectric constant                         | no unit             | no dimensions          |
| 32. | Resistance                                  | $\Omega$            | $[ML^2T^{-3}A^{-2}]$   |

**Example 2:** If velocity (V), force (F) and time (T) are chosen as fundamental quantities, express (i) mass and (ii) energy in terms of V, F and T

**Solution :** Let  $M = (\text{some Number}) (V)^a (F)^b (T)^c$   
Equating dimensions of both the sides  
 $M^1 L^0 T^0 = (1) [L^1 T^{-1}]^a [M^1 L^1 T^{-2}]^b [T^1]^c$   
 $M^1 L^0 T^0 = M^b L^{a+b} T^{-a-2b+c}$   
get  $a = -1, b = 1, c = 1$   
 $M = (\text{Some Number}) (V^{-1} F^1 T^1)$   $[M] = [V^{-1} F^1 T^1]$   
Similarly we can also express energy in terms of V, F, T  
Let  $[E] = [\text{some Number}] [V]^a [F]^b [T]^c$   
 $\Rightarrow [MLT^{-2}] = [M^0 L^0 T^0] [LT^{-1}]^a [MLT^{-2}]^b [T]^c$   
 $\Rightarrow [M^1 L^1 T^{-2}] = [M^b L^{a-2b+c} T^{-a-2b+c}]$   
 $\Rightarrow 1 = b; 1 = a - 2b + c; -2 = -a - 2b + c$   
get  $a = 1; b = 1; c = 1$   
 $E = (\text{some Number}) V^1 F^1 T^1$  or  $[E] = [V^1][F^1][T^1]$ .

### Practice Paper

- Can there be a physical quantity which has no units and no dimensions?
- Can a quantity have dimensions, but still have no units?
- Does the magnitude of a quantity depend on the system of units used?
- Name three physical quantities which have same dimensions.
- A coach told his team, "Muscles times speed is power". What are the dimensions of muscles according to the coach.
- E, m, L, G denote energy, mass, angular momentum and gravitational constant respectively. The dimensions of  $\frac{EL^2}{m^5 G^2}$  will be that of
- If velocity V, force F and energy E are taken as fundamental units, then what is dimensional formula for mass?
- If the force, length and time are taken as fundamental quantities, what will be the dimensions of mass?
- Suppose the velocity of light (c) acceleration due to gravity (g) and pressure (p) are taken as fundamental units. What will be the dimensional formula for mass in this system of units?
- Write two quantities which has units but no dimensions

### Answers

- Yes
- No
- Yes
- Work, Energy, Moment of force
- $M^1 L^1 T^{-2}$
- $M^0 L^0 T^0$
- $EV^{-2}$
- $FL^{-1} T^2$
- $M = pg^{-3} c^4$
- Angular displacement, Sound intensity level

### Uses of dimensions

It can be used to verify the dimensional correctness of a physical equation.



The magnitudes of physical quantities may be added together or subtracted from one another only if they have the same dimensions. In other words, we can add or subtract similar physical quantities. Thus, mass cannot be added to velocity or an electric current cannot be subtracted from time. This simple principle is called the principle of homogeneity of dimensions. This principle is used in checking the correctness of an equation, and also in deducing relation amongst physical quantities. According to principle of homogeneity of dimensions, only that Equation is correct, in which the dimensions of the base quantity on one side of the Equation are equal to the respective dimensions of base quantity on the other side of the Equation.

To check the correctness of the given relation, we shall write the dimensions of the quantities on both sides of the relation. If the principle of homogeneity of dimensions is obeyed, the formula is dimensionally correct.

That mere dimensional correctness of an equation does not ensure its physical correctness. For example, work = torque, is dimensionally correct, but not physically correct.

**Example 3 :** Check the correctness of the relation  $t = 2\pi\sqrt{\ell/g}$  where  $\ell$  is length and  $t$  is time period of a simple pendulum;  $g$  is acceleration due to gravity.

**Solution :** Let us write the dimensions of various quantities on the two sides of the given relation:

$$L.H.S. = t = [T]$$

$$R.H.S. = 2\pi\sqrt{\ell/g} = \sqrt{\frac{L}{LT^{-2}}} \quad (\text{As } 2\pi \text{ has no dimensions})$$

$$= \sqrt{T^2} = [T] \text{ As } L.H.S. = R.H.S., \text{ dimensionally, therefore the given Equation is dimensionally correct.}$$

**Note : 1** Only those physical quantities can be added or subtracted from each other which have the same dimensions. For example, distance can be added or subtracted from distance only and not from any other quantity. Thus  $L + L = L$  and  $L - L = L$

i.e. when one distance is subtracted from some other distance, what is left is some distance only, whatever be its magnitude. Value of distance is irrelevant because we are considering dimensions only. That is why we do not write  $L + L = 2L$  and  $L - L = 0$ .

2. If an equation fails this consistency test, it is wrong. But if the equation passes this consistency test, it is not necessarily right. Thus a dimensionally correct equation may not actually be a physically correct equation. But a dimensionally incorrect/inconsistent equation is a physically incorrect equation.

**Example 4 :** Verify the correctness of the equation  $v = u + at$

$$\text{Dimensions of } v = [LT^{-1}]$$

$$\text{Dimensions of } u = [LT^{-1}]$$

$$\text{Dimensions of } at = [LT^{-2} \times T] = [LT^{-1}]$$

Hence, this equation is dimensionally correct.

**Example 5 :** Check the correctness of the relation  $v^2 - u^2 = 2as$ , where the symbols have their usual meaning.

**Solution :** The given relation is  $v^2 - u^2 = 2as$

Writing the dimensions on either side, we get,

$$L.H.S. = v^2 - u^2 = [LT^{-1}]^2 - [LT^{-1}]^2$$

$$= [L^2 T^{-2}] - [L^2 T^{-2}] = [L^2 T^{-2}]$$

$$R.H.S. = 2as = [LT^{-2}] [L] \quad [\because 2 \text{ has no dimensions}]$$

$$= [L^2 T^{-2}]$$

As  $L.H.S. = R.H.S.$ , dimensionally  $\Rightarrow$  Equation is dimensionally correct.

**Example 6 :** Check the accuracy of the relation  $v = \frac{1}{2l} \sqrt{\frac{T}{m}}$ , where  $v$  is the frequency.  $\ell$  is length,  $T$  is tension and  $m$  is mass of unit length of the string.

$$\text{Solution : The given relation is } v = \frac{1}{2l} \sqrt{\frac{T}{m}}$$

Writing the dimensions on either side, we get

$$\text{L.H.S.} = v = [T^{-1}] = [M^0 L^0 T^{-1}]$$

$$\text{R.H.S.} = \frac{1}{2l} \sqrt{\frac{T}{m}}$$

As  $1/2$  has no dimensions, tension is force and  $m = \text{mass} / \text{length}$

$$\therefore \text{R.H.S.} = \frac{1}{L} \sqrt{\frac{MLT^{-2}}{ML^{-1}}} = \frac{1}{L} \sqrt{L^2 T^{-2}} = \frac{1}{L} (LT^{-1}) = T^{-1} = [M^0 L^0 T^{-1}]$$

As L.H.S. = R.H.S. dimensionally  $\Rightarrow$  Equation is dimensionally correct.

**Example 7 :** Check the correctness of the relation,  $S_{nth} = u + \frac{a}{2}(2n-1)$ , where  $u$  is initial velocity,  $a$  is acceleration and  $S_{nth}$  is the distance traveled by the body in  $n$ th second.

**Solution :** The given relation is  $S_{nth} = u + \frac{a}{2}(2n-1)$

Writing the dimensions on either side, we get

$$\text{L.H.S.} = S_{nth} = \frac{\text{distance}}{\text{time}} = \frac{L}{T} = LT^{-1} = [M^0 L^1 T^{-1}]$$

$$\text{R.H.S.} = u + \frac{a}{2}(2n-1) = LT^{-1} + LT^{-2}(T) = LT^{-1} + LT^{-1} = LT^{-1} = [M^0 L^1 T^{-1}]$$

As LHS = RHS, dimensionally  $\Rightarrow$  Equation is dimensionally correct.

**Example 8 :** In Vander Wall's equation  $\left(P + \frac{a}{V^2}\right)(V-b) = RT$ . What are the dimensions of  $a$  and  $b$ ? Here,  $P$  is pressure,  $V$  is volume  $T$  is temperature and  $R$  is gas constant.

**Solution :** The given equation is  $\left(P + \frac{a}{V^2}\right)(V-b) = RT$

As pressure can be added only to pressure therefore,  $a/V^2$  represents pressure  $P$

$$\text{i.e., } \frac{a}{V^2} = P$$

$$a = PV^2$$

$$a = (ML^{-1}T^{-2})(L^3)^2 = [M^1 L^5 T^{-2}]$$

Again, from volume  $V$ , we can subtract only the volume. Therefore,  $b$  must be representing volume only i.e.

$$b = V = [L^3] = [M^0 L^3 T^0]$$

**Example 9 :** Write the dimensions of  $a$  and  $b$  in the relation  $P = \frac{b-x^2}{at}$ . Where  $P$  is power,  $x$  is distance and  $t$  is time.

**Solution :** The given relation is  $P = \frac{b-x^2}{at}$

As  $x^2$  is subtracted from  $b$ , therefore, the dimensions of  $b$  are  $x^2$  i.e.  $b = [L^2]$

We can rewrite relation as

$$P = \frac{L^2}{at}$$

$$\text{or } a = \frac{L^2}{Pt} = \frac{L^2}{[ML^2T^{-3}][T]} \Rightarrow a = [M^{-1}L^0T^2]$$

**Example 10 :** Find dimension  $x$  and  $y$  of  $\sin(xt^2 - yt)$

**Solution :**  $xt^2 - yt$  is an angle hence dimensionless.

Each term should be dimensionless.

So dimension of  $[x] = [T^2]$  and  $[y] = [T^{-1}]$

**B. Method of dimensions can be used to derive relationship between different physical quantities.**

Using the same principle of homogeneity of dimensions, we can derive the formula of a physical quantity, provided we know the factors on which the physical quantity depends.

We suppose the dimensions of the given physical quantity in terms of these factors, combine them to form an equation; write the dimensions of various quantities in terms of mass, length and time on either side of the equation. Using principle of homogeneity of dimensions, equate the powers of M, L and T on both sides of the dimensional equation. The three equations so obtained are solved to obtain the values of three unknown powers/dimensions. On substituting these values in the equation we formed, we obtain the preliminary form of the relation/equation.

**Example 11:** Derive an expression for time period ( $t$ ) of a simple pendulum, which may depend upon : A mass of bob ( $m$ ), length of pendulum ( $\ell$ ) and acceleration due to gravity ( $g$ ).

**Solution :** Let  $t \propto m^a \ell^b g^c$

where  $a, b, c$  are the dimensions.

or  $t = k m^a \ell^b g^c$

where  $k$  is dimensionless constant of proportionality.

Writing the dimensions in terms of M, L, T on either side of, we get

$$[M^0 L^0 T^1] = M^a L^b (LT^{-2})^c$$

$$= M^a L^{b+c} T^{-2c}$$

Applying the principle of homogeneity of dimensions, we get

$$a = 0, \quad b + c = 0, \quad -2c = 1$$

Solving  $c = -b$  or  $b = \frac{1}{2}$

Putting the values of  $a, b, c$ , we get

$$t = k m^0 \ell^{1/2} g^{-1/2}$$

$$t = k \sqrt{\frac{\ell}{g}}$$

Experimentally  $k$  is found to be  $k = 2\pi \Rightarrow t = 2\pi \sqrt{\ell/g}$

**Example 12 :** Assuming that mass  $m$  of largest stone that can be moved by a flowing river depends upon the velocity,  $v$  of water, its density  $d$ , and acceleration due to gravity  $g$ . Find dimensionally a relation for  $m$ .

**Solution :** Let relation be  $m = k v^a d^b g^c$

Taking dimensions of both sides

$$[ML^0 T^0] = [LT^{-1}]^a [ML^{-3}]^b [LT^{-2}]^c$$

$$ML^0 T^0 = L^a T^{-a} M^b L^{-3b} L^c T^{-2c}$$

$$ML^0 T^0 = M^b L^{a-3b+c} T^{-a-2c}$$

Equating powers of M, L, T  $b = 1$

---- (i)

$$a - 3b + c = 0$$

---- (ii)

$$-a - 2c = 0$$

---- (iii)

On solving,  $a = 6, b = 1, c = -3$

So relation  $m = k v^6 d g^{-3}$

**Example 13 :** The frequency of vibration ( $\nu$ ) of a string may depend upon length ( $l$ ) of the string, tension ( $T$ ) in the string and mass per unit length ( $m$ ) of the string. Using the method of dimensions, derive the formula for  $\nu$ .

**Solution :** Let  $\nu = K l^a T^b m^c$  ..... (i)

where  $K$  is dimensionless constant of proportionality and  $a, b, c$  are the powers of  $l, T$  and  $m$  respectively to represent  $\nu$ .

The tension  $T$  stands for force whose dimensions are  $[M^1 L^1 T^{-2}]$  and  $m = \frac{\text{mass}}{\text{length}} = \frac{M}{L} = [M^1 L^{-1}]$

Writing the dimensions in (i), we get



$$[M^0 L^0 T^{-1}] = L^a (M^1 L^1 T^{-2})^b (ML^{-1})^c$$

$$= L^a M^b L^b T^{-2b} M^c L^{-c}$$

$$[M^0 L^0 T^{-1}] = M^{b+c} L^{a+b-c} T^{-2b}$$

Applying the principle of homogeneity of dimensions, we get

$$b + c = 0$$

$$a + b - c = 0 \quad \dots (ii)$$

$$-2b = -1 \text{ or } b = \frac{1}{2} \quad \dots (iii)$$

from (ii),  $c = -b = -\frac{1}{2}$

from (iii),  $a + \frac{1}{2} - \left(-\frac{1}{2}\right) = 0$

$$a + \frac{1}{2} + \frac{1}{2} = 0 \text{ or } a = -1.$$

Putting these values in (i), we get  $V = K L^{-1} T^{1/2} m^{-1/2}$  or  $v = \frac{K}{l} \sqrt{\frac{T}{m}}$

#### Drawbacks of dimensional analysis in deriving relationship between different physical quantities.:

1. By this method, one cannot find the magnitude of dimensionless constant and gets no idea about dimensional constant.
2. It is not applicable if the relations involve trigonometric, exponential or logarithmic functions.
3. It is difficult to guess the parameters (variables) on which the quantity of interest may depend.
4. If numbers of variables are more than number of equations then exact form of physical relation cannot be derived.

### Practice Paper

1. If  $P = bt + \frac{c+d}{t}$ , where  $P$  = pressure,  $t$  = time, then what is the dimensions of  $c$ ?
2. The velocity  $v$  of a particle at any instant of time  $t$  is given by  $v = at + \frac{b}{t+c}$ . Obtain the dimensions of  $a$ ,  $b$  and  $c$ .
3. Given that  $y = a \cos\left(\frac{t}{p} - qx\right)$ , where  $t$  represent time in sec and  $x$  and  $y$  represents distance in metre. Write dimensions of  $a$ ,  $p$ ,  $q$ .
4. If the unit of force becomes 3 times, length becomes  $\frac{1}{2}$  times and time becomes 2 times, the unit of power will change
5. Find the unit of length, mass and time, if the unit of force, velocity and energy, respectively are 100 dyne, 10 cm  $s^{-1}$  and 500 erg.
6. Calculate  $x$  in the equation  $(\text{velocity})^x = (\text{pressure diff.})^{3/2} \times (\text{density})^{-3/2}$
7. In a certain problem in fluid mechanics it is found that surface tension  $T$  is an important factor. The other significant variables in the problem are velocity  $V$ , force  $F$ , density  $d$  and cross-sectional area  $A$ . Using dimensional methods, derive an expression for force in terms of the other quantities. Assume force is directly proportional to square of surface tension.
8. Force  $F$  is given in terms of time  $t$  and distance  $x$  by  

$$F = A \sin Ct + B \cos Dx$$
  
 Then find the dimensions of  $\frac{A}{B}$  and  $\frac{C}{D}$ .

9. A quantity  $X$  is given by  $\epsilon_0 L \frac{\Delta V}{\Delta t}$ , where  $\epsilon_0$  is the permittivity of free space,  $L$  is a length,  $\Delta V$  is a potential difference and  $\Delta t$  is a time interval. The dimensional formula for  $X$  is the same as that of:
- a) resistance                      b) charge                      c) voltage                      d) current
10. If velocity ( $V$ ), acceleration ( $A$ ) and force ( $F$ ) are taken as fundamental quantities instead of mass ( $M$ ), length ( $L$ ) and time ( $T$ ), the dimensions of Young's modulus would be
- a)  $FA^2V^{-2}$                       b)  $FA^2V^{-3}$                       c)  $FA^2V^{-4}$                       d)  $FA^2V^{-5}$

### Answers

1.  $ML^{-1}T^{-1}$                       2.  $a = LT^{-2}$   $c = T$ ,  $b = L$  3.  $a = L$ ,  $P = T$ ,  $q = L^{-1}$                       4.  $3/4$  times
5.  $5 \text{ cm}$ ,  $\frac{1}{2} \text{ s}$ ,  $5 \text{ gm}$                       6. 3                      7.  $F = \frac{kT^2}{dV^2}$                       8.  $M^0L^0T^0$ ,  $M^0L^1T^{-1}$                       9. d                      10. c

**ALL THE BEST**

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